



Comparison of Simulation Techniques: A Linear Mixed Model Approach

Yabebal Ayalew¹, M.K. Sharma^{1*}, Aynalem Haile² and Meseret Molla³

¹*Department of Statistics, College of Natural Sciences, Addis Ababa University, Addis Ababa, Ethiopia*

²*International Livestock Research Institute, Addis Ababa, Ethiopia*

³*Department of Statistics, Gondar University, Gondar, Ethiopia*

Received 31 August 2010; Revised 02 September 2014; Accepted 29 October 2014

SUMMARY

Most of agricultural data are collected for a long period of time and need high attention. If one record has not been recorded, then the data will become incomplete. In Ethiopia, agricultural researchers have often been challenged by incomplete data. Different simulation techniques with different approximation capability have been used to solve this problem. As a result, this study is aimed to compare which computer based simulation techniques approximate the results of the previously accomplished researches of milk production traits. 15 years of data from *Debre Zeit* Research Station of the International Livestock Research Institute and *Holetta* Agricultural Research Centre of the Ethiopian Institute of Agricultural Research have been used for this study. We compared the two most familiar simulation techniques namely Monte Carlo and bootstrap simulations by using the results of linear mixed model fitted for each dataset. We found that both Monte Carlo and bootstrap simulations can approximate the farm and genetic group effects equally. Lactation length and daily milk yield are found to be significant ($P < 0.0001$) in both simulation techniques. Unlike for bootstrap simulation, season and period of calving are found to be significant for Monte Carlo simulation. On the basis of the findings, this study reached a conclusion that Monte Carlo simulation has a better approximation.

Keywords: Monte Carlo, Bootstrap, Simulation, Model, Linear mixed model, Milk yield.

1. INTRODUCTION

Ethiopia, after getting the first batch of dairy cattle through the United Nations Relief and Rehabilitation Administration, has established seven modern dairy research centers which are distributed in four agro-ecological zones (Alemu *et al.* 1998, Ofcansky and Berry 1991, Hizkias 1998). Even though Ethiopia has large livestock population in the highland region, it has not been getting the desired benefit from them (Hizkias 1998, Muskasa-Mugerwa *et al.* 1989, Saxena 1997).

In order to meet the ever increasing demand of milk and milk products, continuous research must be done on those factors which affect lactation milk yield such as breed type, number of parity, season of calving,

geographic region, farm management factors (nutrition, frequency of milking) age and body weight at calving, season of calving and so forth (Wood 1969, Danell 1982, Wilmlink 1987). One of the problems that prevent researchers from conducting research is that there is no adequate and complete data and database system in the country regarding milk yield. The data which are available and recorded in different research centers are not that much satisfactory. Therefore, we must find alternative statistical methods which can use these incomplete datasets efficiently and reach at valid conclusion.

Different simulation techniques have been used to generate complete datasets based on the given incomplete datasets. However, since the principles that

each simulation method used are different, results emanate from the simulated data may vary. Moreover, to the best of the researchers' knowledge, there has been no research on simulation that is applied on the investigation of problems in lactation milk yield in Ethiopia. Therefore, the objective of this study is to compare which computer based simulation technique better approximate the results of previously accomplished researches regarding milk production traits. Moreover, this paper has two specific objectives. These are comparisons of fixed effect estimates: Examining the significance of fixed effects as well as the coefficients of covariates and contrast testing: Examining the significance of genetic group and herd (farm).

2. MATERIAL AND METHODS

2.1 Data Source

Data for this study were obtained from experimental dairy cattle herds of Ethiopian *Boran* and Ethiopian *Boran-Holstein* crossbred cattle maintained at the *Debre Zeit* Research Station of the International Livestock Research Institute (ILRI) and *Holetta* Agricultural Research Centre of the Ethiopian Institute of Agricultural Research (EIAR). Five genetic groups (Ethiopian *Boran* (0%), 50%, 62.5%, 75% and 87.5% of Holstein inheritance) located from the two farms were used. Fifteen years (1990 to 2004) of data were used for this study. The data has 2594 parities. We defined incomplete dataset as a dataset which doesn't contain the full records of the majority of cows in the farm. For instance, if a cow has delivered 9 calves and the record of a cow indicates only 1st and 5th calves, then we label the record for this cow incomplete. Incompleteness differs from missing data.

The two farms are located at different agro-ecological zone with significantly different rainfall, temperature, humidity and different farm management practices (Alemu *et al.* 1998; Haile *et al.* 2009; Wikipedia 2014).

Monte Carlo and bootstrap simulation techniques were considered in this study and both need computer programming. A general-purpose programming language called C++ was used to translate the simulation model to computer understandable code.

2.2 Monte Carlo Simulation

Monte Carlo simulation is a scheme of employing random numbers from a uniform distribution (Law 2007). Constructing simulation model, selecting input probability distribution and selecting the best algorithm from which we generate the desired data are needed to generate data using Monte Carlo simulation. In order to select the best input probability distribution, we must go through three important steps. First, deciding what general families appeared to be appropriate on the basis of their shapes without worrying about the specific

Table 1. Variable description

Variables	Description	Type of effect	
		Fixed	Random
Genetic group	Classified based on the inheritance of Holstein. These are Ethiopian Boran, 50%, 62.5%, 75% and 87.5% of Holstein inheritance.		✓
Farm	Two farms--- <i>Holetta</i> and <i>Debre Zeit</i>		✓
Parity	The number of live born children delivered	✓	
Season of calving	A period of the year marked by the pattern of the annual rainfall distribution in the area (November to February: dry period; March to June: light rain and July to October: main rainy season).	✓	
Period of calving	Grouped in to five classes: 1990-1992, 1993-1995, 1996-1998, 1999-2001 and 2002-2004	✓	
Lactation length	The number of days in milk	✓	
Lactation milk yield*	The total milk yield in the lactation period		✓
Daily milk yield	The average milk yield per a day	✓	

*The dependent variable

parameter values for these families by examining the shape of the histogram, box plot and results of summary statistics. The second is estimation of parameters of the hypothesized distribution families. We considered explicitly only one type, maximum-likelihood estimator (MLE). The MLE, based on the maximum likelihood function, is appropriate for a variety of parametric distributions since the theoretical probability distributions considered in this study satisfy regularity conditions (Law 2007, Frey and Burmaster 1999, Mood *et al.* 1974). The third step is selecting the best probability distribution from the proposed distributions. We checked the goodness-of-fit by Kolmogorov-Smirnov one-sample, Anderson-Darling and χ^2 goodness-of-fit tests. The process of selecting the best input distribution was done by distribution-fitting software called EasyFit[®](MathWave Technologies 2013). To simulate the values of the above stated variables for a single cow, we used 13 distributions for different scenarios. To generate a single value for any variable, we have considered the possible factors. For instance, to generate daily milk yield, we took the genetic group, fame location, season of calving, parity and lactation length of a cow into consideration. We used the standard algorithms to generate data from these distributions (Rubinstein 1981). The algorithms were:

Algorithm 1. generating Bernoulli random variate

1. Generate $U \sim U(0, 1)$
2. If $U \leq p$, return $X = 1$. Otherwise, return $X = 0$

Algorithm 2. generating Normal random variate (Polar Method)

1. Generate U_1 and $U_2 \sim U(0, 1)$ independently
2. Calculate $W_1 = 2U_1 - 1$ and $W_2 = 2U_2 - 1$
3. If $W = W_1^2 + W_2^2 > 1$ go to step 1
4. Calculate $C = \sqrt{-\frac{2 \ln W}{W}}$, and return $X = CW_1$ and $Y = CW_2$

Algorithm 3. generating Johnson SB random variate

1. Generate $Z \sim N(0, 1)$
2. Let $Y = \exp\left[\frac{(z - \alpha_1)}{\alpha_2}\right]$

3. Return $X = \left[\frac{(a + bY)}{(Y + 1)}\right]$

Algorithm 4. generating Log-Logistic with three parameters

1. Generate $U \sim U(0, 1)$
2. Return $X = \beta \left(\frac{u}{1-u}\right)^{1/\alpha} + \gamma$

Algorithm 5. sampling from the *Gamma* ($\alpha, 1$) distribution ($\alpha \geq 1$)

1. Constants: $a = \alpha - 1$,
 $b = (\alpha - (6\alpha)^{-1})/a$, $c = 2/a$, $d = c + 2$
2. Generate independent $U(0, 1)$ variates U_1 and U_2
3. Let $W = bU_1/U_2$.
If $cU_2 - d + W + W^{-1} \leq 0$ go to step 5
4. If $c \log U_2 - \log W + W - 1 \geq 0$ go to step 2
5. Return $X = aW$

Algorithm 6. sampling from the *Gamma*($\alpha, 1$) distribution ($0 < \alpha < 1$)

1. Compute $b = (e + \alpha)/e$ beforehand
2. Generate $U_1 \sim U(0, 1)$ and let $P = bU_1$. If $P > 1$ go to step 4
3. Let $Y = P^{1/\alpha}$ and generate $U_2 \sim U(0, 1)$. If $U_2 \leq e^{-Y}$, return $X = Y$. Otherwise, go back to step 2
4. Let $Y = -\ln[(b - P)/\alpha]$ and generate $U_2 \sim U(0, 1)$. If $U_2 \leq Y^{\alpha-1}$, return $X = Y$. Otherwise, go back to step 2.

Algorithm 7. generating Pearson type VI of four parameters variate

1. Generate $Y_1 \sim \text{Gamma}(\alpha_1, \beta)$ and $Y_2 \sim \text{Gamma}(\alpha_2, 1)$ independent of Y_1
2. Return $X = (Y_1/Y_2) + \gamma$.

Algorithm 8. generating Dagum with three parameters random variate

1. Generate $U \sim U(0, 1)$
2. Return $X = \beta \left[\left(\frac{1}{U}\right)^{1/k} + 1\right]^{-\frac{1}{\alpha}}$

Algorithm 9. generating Dagum with four parameters random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ Return } X = \beta \left[\left(\frac{1}{U} \right)^{1/k} + 1 \right]^{\frac{1}{\alpha}} + \gamma$$

Algorithm 10. generating Cauchy random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ Return } X = \sigma \tan \left(\pi \left(U - \frac{1}{2} \right) \right) + \mu$$

Algorithm 11. generating Gumbel Min random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ Return } X = \sigma \ln \left(\ln \left(\frac{1}{1-U} \right) \right) + \mu$$

Algorithm 12. generating Laplace random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ If } x \leq \mu, \text{ then return } X = -\frac{\ln(2u)}{\lambda} + \mu;$$

$$\text{otherwise, return } X = -\frac{\ln(2(1-u))}{\lambda} + \mu$$

Algorithm 13. generating Hypersecant random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ Return } X = \frac{2\sigma \tan \left(\frac{\pi u}{2} \right)}{\pi} + \mu$$

Algorithm 14. Generating Gen. Pareto random variate

1. Generate $U \sim U(0, 1)$

2. If $k = 0$, return $X = \sigma \ln(1 - U) + \mu$;

$$\text{otherwise, return, } X = \frac{\left[1 - \left(\frac{1}{1-U} \right)^k \right] \sigma}{k} + \mu$$

Algorithm 15. generating Gen. Extreme value random variate

1. Generate $U \sim U(0, 1)$

$$2. \text{ If, } k \neq 0, \text{ return } X = \frac{\sigma}{k} \left[\left(-\frac{1}{\ln U} \right)^k - 1 \right] + \mu;$$

$$\text{otherwise, return } X = \sigma \ln \left(-\frac{1}{\ln U} \right) + \mu$$

2.3 Bootstrap Simulation

As defined by Efron and Tibshirani (1993), bootstrap simulation is based upon drawing multiple random samples, each of size n , with replacement, from an empirical distribution F . This approach is referred as re-sampling. Each random sample of size n is referred as a bootstrap sample.

Among the four variants of bootstrap re-sampling methods, the nonparametric one was used in this study. The contributions of Efron (1982, 1983) have realized the possibility of simulation without parametric models. Different authors have been trying to improve the efficiency of simulation in Efron's nonparametric bootstrap method. One idea advanced was that of performing a *balanced bootstrap simulation*; that is, one that reuses each of the sample observations exactly equally often. Davison *et al.* (1986) demonstrated that such balancing can yield sizable gains in terms of bias and variance reduction over the usual bootstrap.

Let the sample be denoted by $Y = (Y_1, Y_2, \dots, Y_n)$, $K = \{1, 2, \dots, n\}$ is the index of its elements and B be the number of bootstrap replications. Suppose also that $Rand(a, b)$ is a function that returns a pseudorandom integer, uniformly distributed on $[a, b]$. An algorithm will be described in pseudo code, with the symbols \leftarrow and \leftrightarrow denoting assignment and exchange of two values, respectively.

The balanced bootstrap forces l ($l = j, j \in K$) to assume each of its values exactly B times during the nB replications. Davison *et al.* (1986) pointed out that the balanced bootstrap is, in principle, easy to execute: Concatenate B copies of K to form a list L of length nB . Then randomly permute L and return as bootstrap sample $\{Y_j\}$ corresponding to successive sets of n integers from L .

Algorithm BB1

1. Form the list $L = \{K, K, \dots, K\}$
 2. For $i \leftarrow n * BDOWNTO 2$ DO
 - $j \leftarrow Rand(1, i);$
 - $L_j \leftrightarrow L_i;$
- END FOR

2.4 How Many Replications?

The big concern after the program of simulation written is answering how many replications are needed to have a good estimator and good approximation. The required number of replications (N), given the results of m initial replications is presented as follows:

$$N(m) = \left(\frac{S(m)t_{m-1, \alpha/2}}{\bar{X}(m)\varepsilon} \right)^2$$

where $N(m)$ is the number of replications required, given m replications, $\bar{X}(m)$ is the estimated mean from m simulation run, $S(m)$ is the estimated standard deviation from m simulation run and ε is allowable percentage error of the estimate $\bar{X}(m)$.

Some additional simplifications are used to calculate a fixed number $N(m)$ based on initial estimates of \bar{X} and S , namely, the initial estimates $\bar{X}(m)$ and $S(m)$ based on m observations are close enough to the real mean and standard deviation, and thus do not change much when the number of replications is increased to $N(m)$. By this formula, we generated 3483 parities for Monte Carlo simulation and 3559 parities for bootstrap simulation.

2.5 Linear Mixed Models

A linear mixed model is an abstract representation of the real world by using fixed and random effects as its components. We considered the linear mixed model

$$y = X\beta + Zu + \varepsilon$$

where y is a column vector of N observations, β is a column vector of N_f fixed effects including covariates, X is the $N \times N_f$ design matrix for fixed effects, u is a column vector of N_u random effects, Z is the $N \times N_u$ design matrix for random effects, and ε is the column vector of N errors.

For the (co)variance structure of y the assumptions are, $var(u) = G$, $var(\varepsilon) = R$, $cov(u, \varepsilon) = 0$ and $var(y) = V = ZGZ' + R$.

The farm and genetic group were treated as random effect and the rest were considered as fixed effect. Since the model contains random disturbance and effect, the dependent variable was also treated as random. The (co)variance structure used in this study is variance components. Restricted Maximum Likelihood estimation technique was applied to find the estimate of the coefficients of fixed effects and the realization of random effects. SAS was used to analyze the linear mixed model.

3. RESULTS AND DISCUSSION

After the appropriate model fitting process (top-down strategy) and diagnostic checking (influential analysis, homoscedasticity, normality, etc.) applied on the two generated data sets, the following results were produced using SAS 9.2.

3.1 Fixed Effect Estimates

As can be seen in Table 2 and Table 3, the number of live-born calves has a pronounced effect on the amount of milk that we obtain per lactation period. Keeping the other factors' effect constant, the increment of the offspring of cow by one makes the amount of milk reduced by 1.94 liters.

The effect of season of calving is significant at 5% level of significance. The effect of season of calving on lactation milk yield is confounded by breed (genetic group), the stage of lactation and climatic condition. However, seasonal differences have less significant effect because of better feeding and management of the dairy cow.

Lactation length and daily milk yield have a prominent positive effect on lactation milk yield. Furthermore, by holding the effect of other factors constant, lactation milk yield increased by 5.8837 and 309.13 liters for a unit increment in lactation length and daily milk yield, respectively.

Lactation length and daily milk yield have a prominent positive effect on lactation milk yield. Furthermore, by keeping the effect of other factors constant, lactation milk yield increased by 5.8837 and

Table 2. Fixed effect estimates for Monte Carlo dataset

Effect	Season	Period	Estimate	Standard error	DF	t value	Pr > t
Intercept			-1814.97	102.97	1	-17.63	0.0361
Parity			-1.9362	0.9332	3301	-2.07	0.0381
Season	Light rain		-18.7881	5.2639	7	-3.57	0.0091
Season	Main rain		22.3197	7.6812	7	2.91	0.0228
Season	Dry season		0
Period		1990-92	-3.2845	3.7346	3301	-0.88	0.3792
Period		1993-95	-1.2386	2.8070	3301	-0.44	0.6591
Period		1996-98	-6.9695	2.5403	3301	-2.74	0.0061
Period		1999-01	-3.6514	2.5284	3301	-1.44	0.1488
Period		2002-04	0
Lactation Length			5.8837	0.2630	4	22.37	<0.0001
Daily Milk Yield			309.13	9.3237	4	33.16	0.0003
Parity*Season	Light rain		3.7552	1.0491	3301	3.58	<0.0001
Parity*Season	Main rain		-7.0003	1.2704	3301	-5.51	<0.0001
Parity*Season	Dry season		0

*DF stands for degree of freedom

309.13 liters for a unit increment in lactation length and daily milk yield, respectively.

Lactation milk yield is also affected by the interaction of number of live-born calves and season of calving. That is, when we keep the effect of other factors constant, a unit increment of live-born calf makes the lactation milk yield raised and abridged by 3.76 and 7 liters in light and main rain seasons, respectively. Period of calving is significant at 5% and 10% level of significances (Table 3).

Table 3. Fixed effect tests for Monte Carlo dataset

Effect	Num DF	Den DF	F value	Pr > F
Parity	1	3301	15.35	<0.0001
Season	2	7	7.65	0.0173
Period	4	3301	2.47	0.0429
Lactation length	1	4	500.62	<0.0001
Daily milk yield	1	4	1099.30	<0.0001
Parity*Season	2	3301	34.98	<0.0001

As can be observed from Table 4 and Table 5, parity has no significant effect on lactation milk yield.

In rare cases, under optimal feeding and proper health care management in the farm(s) may make the effect of parity less significant. Nevertheless, we can't snub the effect of parity whether the effect is negative or positive (Haile *et al.* 2009).

Lactation length and daily milk yield have incontrovertible positive effect on the dependent variable, lactation milk yield. The effect of these two determinant factors are expected because increasing the number of days of cow in milk increases the total milk yield in lactation period keeping the frequency and interval of milking in mind.

The two most important environmental factors, season and period of calving, which encompass ambient temperature implicitly, are insignificant. Though the effect of season and period of calving is depending on the type of genetic group, Holsteins and other larger breeds are more resistant to lower temperatures. The optimal temperature for Holstein cow is about 10°C. The milk production declines when an environmental temperature exceeds 27°C. The annual average temperature of *Debre Zeit* and *Holetta* are 18.7°C and 19-24°C, respectively (Haile *et al.* 2009). Since parity and season of calving are not significant, the interaction effect of these two variables becomes insignificant.

Table 4. Fixed effect estimates for bootstrap dataset

Effect	Season	Period	Estimate	Standard error	DF	t value	Pr > t
Intercept			-1456.20	81.3603	1	-17.90	0.0355
Parity			-0.6815	1.2884	3382	-0.53	0.5969
Season	Light rain		-0.5972	4.5141	8	-0.13	0.8980
Season	Main rain		-5.6003	4.4266	8	-1.27	0.2414
Season	Dry season		0
Period		1990-92	-8.0144	3.9104	3382	-2.05	0.0405
Period		1993-95	-3.8829	3.3042	3382	-1.18	0.2400
Period		1996-98	-0.7420	3.0096	3382	-0.25	0.8053
Period		1999-01	-1.3629	2.8501	3382	-0.48	0.6325
Period		2002-04	0
Lactation Length			4.8536	0.2050	4	23.67	<0.0001
Daily Milk Yield			300.46	4.7460	4	63.31	<0.0001
Parity*Season	Light rain		0.2371	1.4653	3382	0.16	0.8715
Parity*Season	Main rain		1.0154	1.4451	3382	0.70	0.4823
Parity*Season	Dry season		0

*DF stands for degree of freedom

Table 5. Fixed effect tests for bootstrap dataset

Effect	Num DF	Den DF	F value	Pr > F
Parity	1	3382	0.08	0.7841
Season	2	8	0.95	0.4253
Period	4	3382	1.30	0.2661
Lactation length	1	4	560.38	<0.0001
Daily milk yield	1	4	4007.88	<0.0001
Parity*Season	2	3382	0.27	0.7626

3.2 Contrast Testing

3.2.1 Farm Effect

The null hypothesis for the two datasets is that *Debre Zeit* and *Holeta* farm have the same average effect on lactation milk yield. In other words, the farm management practices in these two dairy farms have the same effect.

Table 6 exhibits that the farm effect is significant. The farm management practice in *Debre Zeit* and *Holeta* don't have the same effect on lactation milk yield. That is, the farm management practices in these farms are the determinant factors for the difference in the average milk yield per lactation. As Haile *et al.* (2009) confirmed, due to problem of tick infestation,

Table 6. Farm effect test

Dataset	Label	Num DF	Den DF	F value	Pr>F
Monte Carlo	Farm effect	1	4.47	311.26	<0.0001
Bootstrap	Farm effect	1	4.01	323.35	<0.0001

cattle in *Debre Zeit* are not graze. As a result they are stall fed. Teff (*Eragrostis tef*) straw, clean water, hay and mineral lick are provided *ad libitum*. Furthermore, based on milk production, the animals are supplemented with concentrate mixture composed of wheat bran, noug seed cake (*Guizoita abyssinica*) and molasses twice a day.

The herd at *Holeta* is grazing on natural pasture for about 8 hours during daylight. At night all animals are supplemented with natural pasture hay conserved from part of grazing area. Except for the lactating cows, which are supplemented with 3-4kg of concentrate at each milking, no other animal receives any regular concentrate supplement unless animals' condition is deteriorating in the long-dry period.

3.2.2. Genetic Group Effect

The null hypothesis is that the five genetic groups have the same average effect on lactation milk yield.

In other words, the average yield of milk in a lactation period is the same across the five genetic groups or put differently their percentage of Holstein inheritance doesn't have any effect on average yield of milk per lactation period.

Table 7. Genetic group effect

Dataset	Label	Num DF	Den DF	F value	Pr>F
Monte Carlo	Genetic effect	1	38.3	1125.15	<0.0001
Bootstrap	Genetic effect	1	39.6	1450.76	<0.0001

As it is evident from Table 7, all five genetic groups don't have the same effect on the average milk yield per lactation period.

4. CONCLUSION

In the case of Monte Carlo simulation, season and period of calving are significant. The same result was reported in many researches (Goshu 2005, Raheja 1994, Mukerjee 2005). In contrast, season of calving is not significant in case of bootstrap simulation. This result agrees with the work of Haile *et al.* (2009), Gebeyehu *et al.* (2005) and Fayaye and Ayorinde (2010). The work of Haile *et al.* (2009) shows that unlike season of calving, period of calving is significant. Whatever the optimal feeding and management conditions attained there is always the effect of either season or period of calving on lactation milk yield (Goshu 2005, Million and Tadelle 2003).

Unlike bootstrap simulation, parity is significant in case of Monte Carlo simulation. This result agrees with what Gebeyehu *et al.* (2005), Gebeyehu *et al.* (2007) and Haile *et al.* (2009) have reported. In contrast to bootstrap simulation, the interaction effect of parity by season of calving is significant in Monte Carlo simulation. This result was reported by Ray *et al.* (1992).

The lactation length and daily milk yield are significant in both simulation results. The coefficients are almost similar. There is no significant difference between these simulations in this regard (Million and Tadelle 2003).

For contrast testing, both simulation techniques have equal performance. Haile *et al.* (2009) reported

that, due to lack of consistent variation in the two herds, farm effect is insignificant. Many research works confirmed that farm has significant effect on lactation milk yield even under similar and standard farm management practices (Mukerjee 2005, Jadhav *et al.* 1991).

This study shows that there are very good approximations of results in both simulations. In this study, however, the result of bootstrap simulation has neglected environmental effect. In fact, this result is in contrast of what were reported by other researches. In contrast, Monte Carlo simulation produces results which are highly in line with the previously accomplished researches in the field. Therefore, this study reveals that Monte Carlo simulation provides a better approximation as to compare with bootstrap simulation.

ACKNOWLEDGEMENTS

We sincerely thank ILRI and EIAR for providing the data. We also thank referees for their constructive suggestions and comments.

REFERENCES

- Alemu, G., Mengistu, A., Sendros, D. and Alemu, T. (1998). Status of dairy research in Ethiopia. *Proceedings of the Smallholder Dairy Development of Project*, Addis Ababa, Ministry of Agriculture, 73-81.
- Danell, B. (1982). Studies on lactation yield and individual test day yields of swedish red cows. *Acta Agriculturae Scandinavica*, **32**, 65-81.
- Davison, A.C., Hinkley, D.V. and Schechtman, E. (1986). Efficient bootstrap simulation. *Biometrika*, **73**(3), 555-566.
- Efron, B. (1982). The Jackknife, the Bootstrap, and other resampling plans. *Conf. Series in Appl. Math.*, **38**, SIAM, Philadelphia.
- Efron, B. (1983). Estimating the error rate of a prediction rule: improvement on cross-validation. *J. Amer. Statist. Assoc.*, **78**, 316-331.
- Efron, B. and Tibshirani, R.J. (1993). *An Introduction to Bootstrap. Monographs on Statistics and Applied Probability*. Chapman and Hall, New York.
- Fayaye, T.R. and Ayorinde, K.L. (2010). Effects of season, generation, number of mating, parity and Doe number of teat on Doe and litter birth characteristics in domestic Rabbit. *Res. J. Anim. Vet. sci.*, **5**, 6-9.

- Frey, H.C. and Burmaster, D.E. (1999). Methods for characterizing variability and uncertainty: Comparison of bootstrap simulation and likelihood-based approaches, *Risk Analy.*, **19**, 109-130.
- Gebeyehu, G., Asmare, A. and Asseged, B. (2005). Reproductive performances of Fogera cattle and their Friesian crosses in Andassa ranch, Northwestern Ethiopia. *Livestock Res. Rural Dev.*, **17**. <http://lrrd.org/lrrd17/12/gosh17131.htm> [Accessed 16 August 2014].
- Gebeyehu, G., Belihu, K. and Berihun, A. (2007). Effect of parity, season and year on reproductive performance and herd life of Friesian cows at Stella private dairy farm, Ethiopia. *Livestock Res. Rural Dev.*, **19**. <http://lrrd.org/lrrd19/7/gosh19098.htm> [Accessed May 15, 2014].
- Goshu, G. (2005). Breeding efficiency, lifetime lactation and calving performance of Friesian-Boran crossbred cows at Cheffa farm, Ethiopia. *Livestock Res. Rural Dev.*, **17**. <http://www.lrrd.org/lrrd17/7/gosh17073.htm> [Accessed May 15, 2014]
- Haile, A., Joshi, B.K., Ayalew, W., Tegegne, A. and Singh, A. (2009). Genetic evaluation of Ethiopian Boran Cattle and their crosses with Holstein Friesian in central Ethiopia: Milk production traits. *Animal*, **3**, 486-493.
- Hizkias, K. (1998). Dairy development in Ethiopia. *Proceedings of the Smallholder Dairy Development of Project*, Ministry of Agriculture, Addis Ababa, 26-39.
- Jadhav, K.L., Tripathi, V.N., Taneja, V.K. and Kale, M.M. (1991). Performance of various HolsteinSahiwal grades for first lactation reproduction and production traits. *Ind. J. Dairy Sci.*, **44**, 209-215.
- Law, A.M. (2007). *Simulation Modeling and Analysis*. McGraw-Hill, New York.
- MathWave Technologies (2013). EasyFit[®], www.mathwave.com/products/easyfit.html [Accessed 20 March 2013].
- Million T. and Tadelle D. (2003). Milk production performance of Zebu, Holstein Friesian and their crosses in Ethiopia. *Livestock Res. Rural Dev.*, **(15)3**. <http://www.lrrd.org/lrrd15/3/Tade153.htm> [Retrieved 16 August 2014].
- Mukerjee, S. (2005). *Genetic evaluation of Frieswal cattle*. Ph.D, National Research Institute, Karnal, India.
- Mood, A.M., Graybill, F.A. and Boes, D.C. (1974). *Introduction to the Theory of Statistics*. McGraw-Hill, Singapore.
- Muskasa-Mugerwa, E., Bekele, E. and Tessema, T. (1989). Type and productivity of indigenous cattle in central Ethiopia. *Trop. Anim. Hith. Prod.*, **21**, 120.
- Ofcansky, T.P. and Berry, L. (1991). *Ethiopia: A country study*. Washington, GPO for the library of congress.
- Raheja, K.L. (1994). Genetic parameters for first lactation and lifetime production traits in Friesian–Hariana and Friesian–Sahiwal halfbreds estimated by multiple trait maximum likelihood procedures. *Ind. J. Anim. Sci.*, **64**, 616-621.
- Ray, D.E., Halbach, T.J. and Armstrong, D.V. (1992). Season and lactation number effects on milk production and reproduction of dairy cattle in Arizona. *J. Anim. Sci.*, **75**, 2976-2983.
- Rubinstein, R.Y. (1981). *Simulation and Monte Carlo Method*. Wiley, New Jersey.
- Saxena, M.M., Katpatal, B.G. and Pandey, H.S. (1997). Study of milk constituents and their yield in Holstein–Friesian cows. *Ind. J. Anim. Prod. Manage.*, **13**, 127-130.
- Wikipedia (2014). Bishoftu. http://en.wikipedia.org/wiki/Debre_Zeyit [Accessed 16 August 2014].
- Wilmink, J.B.M. (1987). Adjusted of test day milk, fat and protein yield for age, season and stage of lactation. *Livestock Prod. Sci.*, **16**, 321-334.
- Wood, P.D.P. (1969). Factors affecting the shape of the lactation curve in cattle. *Anim. Production*, **11**, 307-316.